BOWLAND
MATHS
Professional development

Tackling unstructured problems

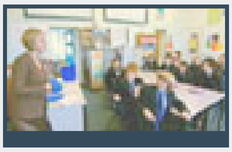
'Do I stand back and watch, or intervene and tell them what to do?'

Module overview

In mathematics lessons, pupils are usually told which prerequisite information they need and which techniques to deploy. If, however, pupils are to learn to use their mathematics autonomously outside the classroom, they also need opportunities to work on unstructured problems that require the selection and use of a wide range of mathematical techniques. This module compares structured and unstructured versions of problems and considers the demands and challenges unstructured problems present to pupils and teachers.

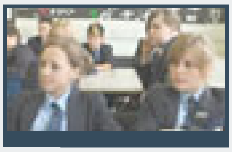
This guide is intended for use alongside the *Bowland Maths DVD* or website, which include a short introductory video for each of the activities; longer videos of lessons and teacher discussions and links to all the handouts and ICT-based problems.

Introductory session — 1 hour



- Critique and revise structured problems
- Compare structured and unstructured problems
- Observe teachers using unstructured problems
- Discuss pedagogical implications
- Plan a lesson using one of the problems

Into the classroom — 1 hour



- Introduce the problem to the class
- Pupils work on the problem
- Pupils share different approaches
- Pupils continue with the problem

Follow-up session — 1 hour

- Report and reflect on the lesson
- Observe teacher interventions
- Consider strategies for offering help
- Discuss how you handle sensitive issues
- Plan some strategies for future lessons

Resources needed

Handout 1	Three structured problems
Software	Body Mass Index Calculator (optional)
Handout 2	Three unstructured versions of the problems
Handout 3	Notes on the unstructured problems
Handout 4	Practical advice for teaching problem solving
Handout 5	Suggested further reading

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Introductory session

Introduction What is an unstructured problem?

In mathematics lessons (and examinations), most of the 'problems' given to pupils are not really problems at all - they are highly structured exercises. Pupils are told which prerequisite information they need and which techniques to deploy. They are then led by a chain of short, closed, questions to the required, unique, solution. Alternative approaches are neither encouraged nor discussed. If, however, pupils are to learn to use their mathematics autonomously outside the classroom, they also need opportunities to work on unstructured problems that require the selection and use of a wide range of mathematical techniques.

In this module, we compare structured and unstructured versions of problems and consider the demands and challenges they present to pupils and teachers. We try out one or two unstructured problems with a Key Stage 3 class and then go on to consider the support pupils need to work on more 'open' problems of this type.

Activity 1 Critique and revise structured problems 10 minutes

Handout 1 presents three structured problems:

- *Organising a table tennis tournament*
- *Designing a box for 18 sweets*
- *Calculating Body Mass Index*

These problems are of the same type as those typically found in many of the Bowland Case Studies. These are, however, structured so that they lead pupils through the problems, guiding and making decisions for them.

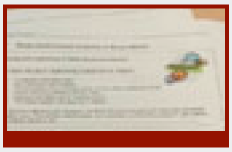
Work through one of the structured problems carefully.

- List all the decisions that are being made for the pupils.
- Revise the problems so that some of these decisions are handed back to pupils. This will make them less structured.

For example, in *Organising a table tennis tournament*, pupils are told:

- how to code the players (A, B, C .. etc)
- to list all the matches that need to be played
- how to systematically organise these matches
- how to tabulate the order of play
- to remember that players cannot play on two tables at once.

Activity 2	Compare structured and unstructured problems	5 minutes



Compare the unstructured versions of the problems (Handout 2) with the structured versions.

- What are the essential differences?
- What pedagogical issues will arise when you start to use unstructured problems like this?

Some immediate issues with the less structured problems are:

- They are more difficult.
- Pupils may not even know how to get started on them.
- If we offer help too quickly, pupils will simply do what we say and not think for themselves.
- They will generate a greater variety of approaches and solutions.
- Pupils may need reassurance that it is OK to try a different approach or reach a different conclusion.

Handout 3 explains the links to the Case Studies and gives possible solution strategies to these problems.

Activity 3	Observe teachers using unstructured problems	20 minutes

The three video clips show pupils working with the unstructured versions of the same problems you have worked on. The first time through, we suggest you watch Michelle using the *Organising a table tennis tournament* problem. You can return to the other clips another time.

As you watch the video, consider:

- How did the teacher organise the classroom? Why were pupils expected to work in pairs/ small groups?
- How did the teacher introduce the problem to pupils?
- What different approaches were being used by pupils?
- How did the teacher support the pupils that were struggling?
- How did the teacher encourage the sharing of approaches and strategies?
- What do you think these pupils were learning?

Activity 4	Discuss pedagogical implications	15 minutes

Watch the three teachers talking about how they will introduce the unstructured problems to their pupils.

- What culture are these teachers trying to create in the classroom?
- How did the teachers plan to make the problem more accessible to students?
- What do they plan for pupils that finish quickly?

Handout 4 offers general practical advice for teaching problem solving:

- Allow pupils time to understand and engage with the problem.
- Offer strategic hints, rather than technical guidance.
- Allow opportunities to compare alternative approaches.
- Place the focus on explaining methods, rather than on obtaining answers.
- Model thinking and powerful methods.

- What aspects of this advice were the teachers considering?
- Discuss the advice and consider the implications.
- Add your own ideas for advice to the bottom of the handout.

Activity 5	Plan a lesson using one of the problems	10 minutes

Choose one of the three problems that you feel would be appropriate for your class.

Discuss how you will:

- Organise the classroom and the resources needed.
- Introduce the problem to pupils.
- Explain to pupils how you want them to work together.
- Challenge/assist pupils that find the problem straightforward/ difficult.
- Help them share and learn from alternative problem-solving strategies.
- Conclude the lesson.

If you are working on this module with a group, it will be helpful if each participant chooses the same problem, as this will facilitate the follow-up discussion.

This is the end of the *Introductory session*. After you have tried out your lesson with your own pupils, return for the *Follow-up session*.

Resources to support the lessons, and suggested lesson plans, can be found in the *Into the classroom* session.

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Into the classroom

Tackling unstructured problems

'Do I stand back and watch, or intervene and tell them what to do?'

The following suggestions describe one possible approach to using the problems with pupils. This may take one or two lessons, depending on the class.

Introduce the problem to the class — 10 minutes

Give each pupil with one of the three problems.
Explain the purpose of the lesson:

The aim of today's lesson is to see how you go about tackling a problem from the real world without my help. This problem is a bit different to problems that you normally see in maths lessons - it's more like a problem from everyday life.

- *You will not be told which bits of maths to use.*
- *There are many ways to tackle the problem - you choose.*
- *There may be more than one 'right answer'.*

I want to see how creative you can be, and see if you can think of a good way to solve the problem. Later in the lesson we will compare the different approaches we have used.

Explain how pupils are expected to work on the problem:

I want you to read and think about the problem on your own for five minutes. Then I'll ask you to share ideas in pairs or threes. From then on, you should work together with your partner(s) to find a good way to tackle the problem.

Describe the resources that are available for working on the problem. Where appropriate, leave these at the side of the room, so that pupils can choose whether or not they use them:

Table tennis tournament
For making your poster we have some large sheets of A3 paper and felt-tipped pens.

Box for sweets
To help you design and make the box, we have some A4 card, scissors, glue sticks, squared and isometric dotted paper and even some sweets available, should you choose to use them!

Body Mass Index Calculator
You will need one computer between two. There is also a supply of calculators, squared paper and graph paper, should you choose to use them.

Explain what pupils should do when they are stuck:

If you get stuck, then don't ask me what to do! I'm not going to do the problem for you. Everyone gets stuck from time to time. You might just need a few more minutes to think more carefully. You might need to experiment with a few ideas. You might need to talk through the problem with your partner. Try to find something in the problem that you can do to get started.

Now set a target:

Right, now I'm giving you twenty minutes to work on the problem by yourselves. Then I'm going to ask some of you to come out and talk about the different approaches you are using.

Pupils work on the task 20 minutes

Allow pupils time to engage with the problems. When they ask questions, offer strategic guidance rather than technical help. Use questions such as those found on Handout 4. For example:

Take you time, don't rush.
What do you know?
What are you trying to do?
What do you need to find out?
What is fixed? What can be changed?
Don't ask for help too quickly - try to think it out between you.

Pupils share different approaches 10 minutes

When most pupils have made significant progress with the problem, invite pairs of pupils to come to the front and share their ideas:

Let's stop and share some of the different approaches we have used and consider what has been helpful and unhelpful about each method.
Not everyone has finished, so I don't want to know about your answers.
I want you to offer your advice for helpful ways to make progress with problems like this.

Table tennis tournament
"We decided to use 7 counters for the different players and we drew little table tennis tables. Then we tried to find a way of moving the players round the tables so that they all played each other."

"We made a list of all the matches that we needed first. Then we tried to place these in a two-way table, so that each person only appears once on each row."

Box for sweets
"We arranged 8 sweets on the table in different ways and sketched different possible boxes that would fit. Then we drew round the sweets on the table to see how big the sides of the box would be. Then we tried to fit the sides together."

"We didn't use sweets at all. We worked out the dimensions of the box and then tried to draw it on squared paper. It was tricky trying to imagine where all the flaps would go."

Body Mass Index Calculator

"We experimented for a bit, then we decided to fix the height at 1 metre and made a table of BMI against weight. We plotted a graph to look for patterns and found that it goes up in equal steps."

"We made a two-way table with height going up the page and weight going across. Then we filled in the BMI values and looked for patterns."

As pupils present their ideas, ask other pupils to comment on the advantages and disadvantages of each approach.

Pupils continue with the problem 20 minutes

Encourage pupils to return to the problem and continue working on it using some of the ideas that have been shared. Collect examples of pupils' work for the follow-up session.

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Follow-up session

Activity 1 | Report and reflect on the lesson | 20 minutes

Take it in turns to share stories of what happened in your lessons. Give factual, descriptive accounts avoiding unhelpful judgements such as 'it went well'.

Now you have taught the lesson, it is time to reflect on what happened.

- What range of responses did pupils have to this way of working? Did some appear confident? Did some need help? What sort of help? Why did they need it?

- What support and guidance did you feel obliged to give? Why was this? Did you give too much or too little help?

- What different strategies did pupils use? Share two or three different examples of pupils' work.

- What do you think pupils learned from this lesson?

If there is time, you may also like to watch the videos of the teachers as they reflect on their own lessons with the *Table tennis* and *Sweet box* problems.

Activity 2 | Observe teacher interventions | 10 minutes

The video shows teachers questioning pupils while they worked on the three problems.

Consider the following issues:

- What kinds of questions were they asking pupils?
- What was the purpose of these questions?
- How do these questions compare with those you asked in your lesson?

Activity 3 Consider strategies for offering help 10 minutes

Pupils unused to working on open problems often seek reassurance from the teacher that they are heading in the right direction: "Is this right?" "What do I do next?"

- What strategies can you use to help pupils to work more autonomously and independently?

- Do you think it is better to 'throw pupils in at the deep end' and give them unstructured problems straight away, or to *gradually* remove the scaffolded support over a period of time? How would you do this?

Activity 4 Discuss how you handle sensitive issues 10 minutes

One of the problems, *A Body Mass Index calculator*, was not used by some teachers because they felt that it might cause embarrassment or disturb pupils who are sensitive about their own bodies. Others felt that such sensitive issues should be tackled head on, and addressed directly.

- Do you feel that sensitive issues should be addressed in the mathematics classroom? Why or why not?
- They are addressed in other subjects, including English and Science. Why are these different?
- Would you feel comfortable using this particular resource in your classroom?
- Would working with a PSHE teacher help?
- Would you make any special arrangements before using such a resource?

The case studies themselves do contain some issues that may be considered sensitive to some pupils. *How Risky is Life?*, for example, considers the likelihood of unexpected death from various causes and this may be sensitive to those who have been recently bereaved by an accident. The PSHE KS3 programme of study specifically says that "*pupils should be taught to recognise and manage risk and make safer choices about healthy lifestyles...*"

Activity 5 Plan strategies for future lessons 10 minutes

Plan some ways of applying what you have learned in this PD module to the other mathematics lessons that you teach.

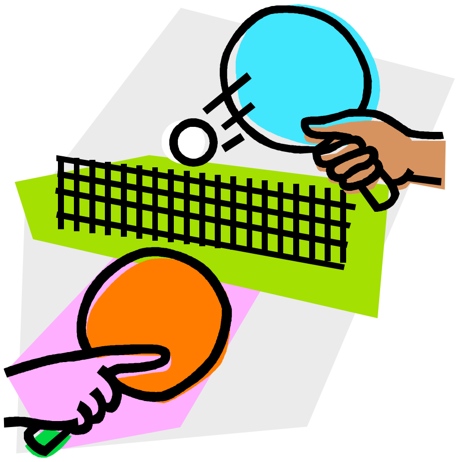
- Take examples of activities that you normally use in mathematics lessons and consider how they may be made less structured.
- Develop 'suggestions' to be given orally or in writing to pupils who demonstrate that they really need more support as they work on unstructured problems.

Further Reading

See Handout 5 for suggested further reading.

1 Three structured problems

Organising a table tennis tournament



You have the job of organising a table tennis tournament.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.

1. Call the players A, B, C, D, E, F, G
   Complete the list below to show all the matches that need to be played.

A v B	B v C	....
A v C	B v D	....
....	....	

2. There are four tables at the club and each game takes half an hour.
   The first match will start at 1.00pm.

   Copy and complete the poster below to show the order of play, so that the tournament takes the shortest possible time.
   Remember that a player cannot be in two places at once!
   You may not need to use every row and column in the table!

Start Time	Table on which the game will be played			
	1	2	3	4
1.00	A v B			
1.30				
2.00				
2.30				
3.00				
3.30				
4.00				
4.30				

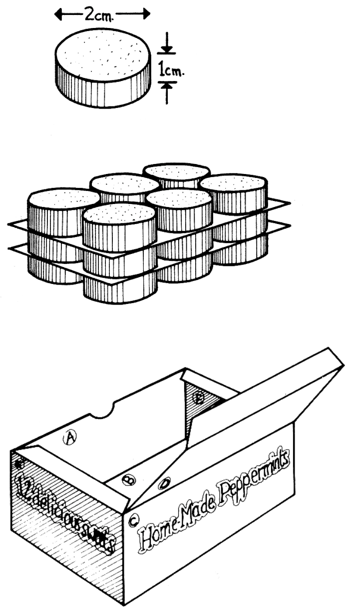
1 Three structured problems (continued)

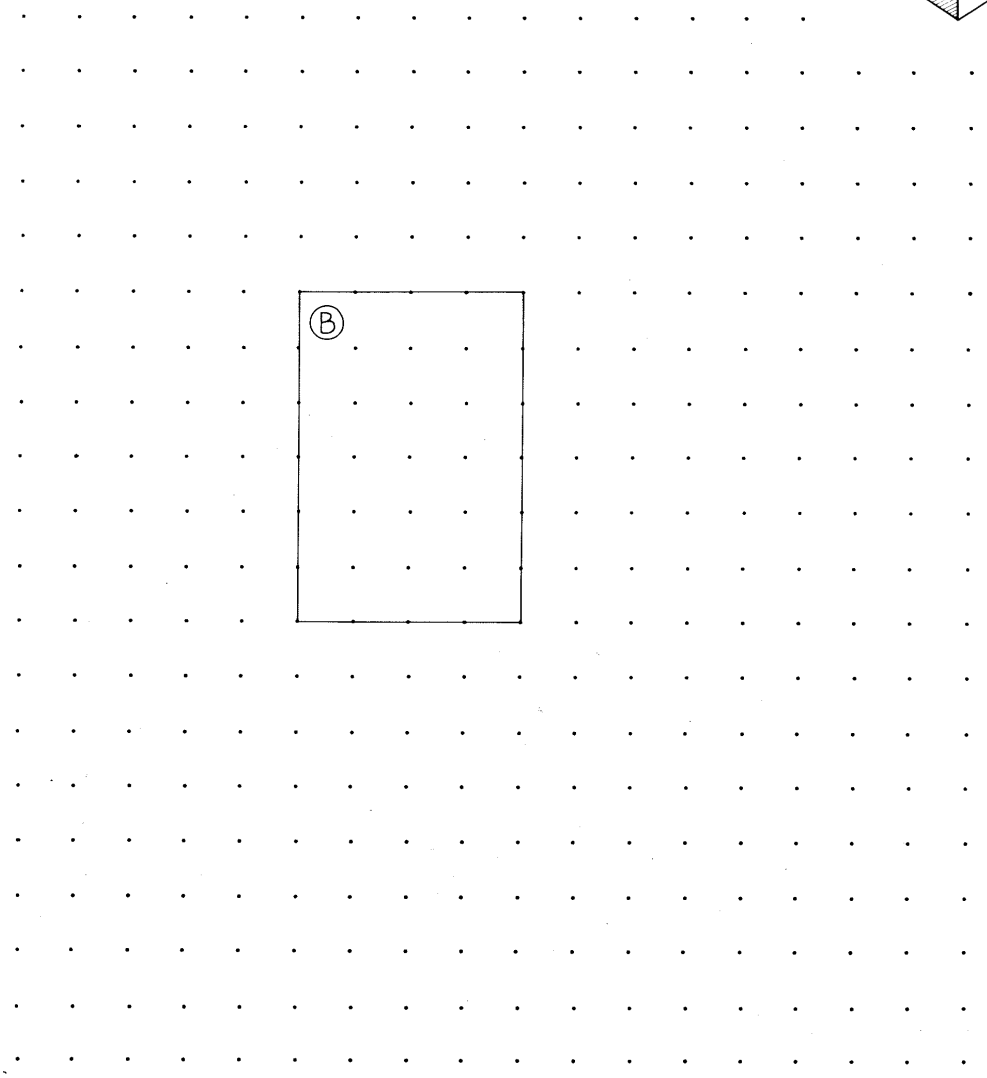
Designing a box for 18 sweets

You work for a design company and have been asked to design a box that will hold 18 mints.
Each mint is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of A4 card with as little cutting as possible.

On the grid paper below, show clearly how the card can be folded up and glued together to make the box.

Make your box to check.

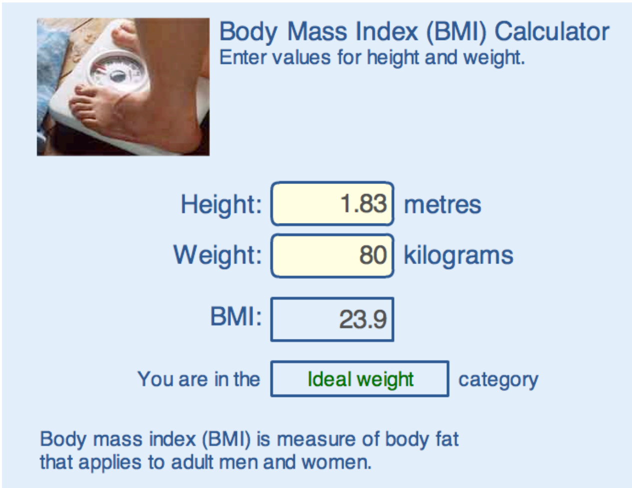




1 Three structured problems (continued)

Calculating Body Mass Index

This calculator is used to help adults find out if they are overweight.



1. Fix the height at 2 metres - a very tall person!
   Complete the table below and draw a graph to show your results.

Weight (kg)	60	70	80	90	100	110	120	130	140
BMI									

   (a) What is the largest BMI for which someone is underweight?
   (b) What is the smallest BMI for which someone is overweight?
   (c) When you double the weight, what happens to the BMI?
   (d) Can you find a rule for calculating BMI from the weight?

2. Fix the weight at 80 kilograms and try varying the height.

   (a) When you double the height, what happens to the BMI?
   (b) Can you find a rule for calculating BMI from the height?
   (c) Draw a graph to show the relationship between the height and the BMI.

For more information on BMI visit:

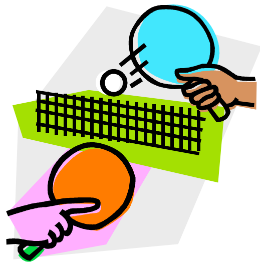
http://www.nhsdirect.nhs.uk/magazine/interactive/bmi/index.aspx

Note for pupils: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

2 Three unstructured versions of the problems

Planning and organising — Organising a table tennis tournament

You have the job of organising a table tennis league.

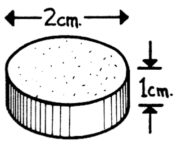


- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

Plan how to organise the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do.

Designing and testing — Designing a box for 18 sweets

You work for a design company and have been asked to design a box that will hold 18 sweets.
Each sweet is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of A4 card with as little cutting as possible.



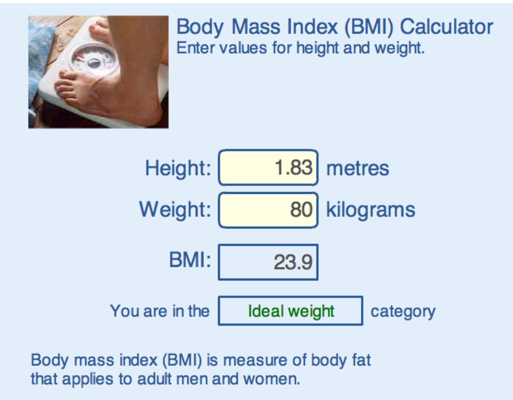
Compare two possible designs for the box and say which is best and why.

Make your box.

Exploring and discovering — Calculating Body Mass Index

This calculator shown is used on websites to help an adult decide if he or she is overweight.
What values of the BMI indicate whether an adult is underweight, overweight, obese, or very obese?

Investigate how the calculator works out the BMI from the height and weight.



Note for pupils: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

3 Notes on the unstructured problems

Planning and organising — **Organising a table tennis tournament**

Links to Case Studies

This problem is concerned with planning and organising. It involves allocating limited resources under realistic constraints in order to find an optimal solution. This is similar to other optimisation problems within the Case Studies, for example:

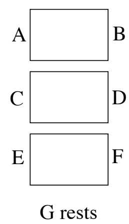
- **Outbreak** where pupils are asked to find ways of mixing ingredients to create an antidote for a virus and devise a vaccination programme.
- **Mystery tours**, where pupils plan a nationwide tour to satisfy time/money/customers
- **Highway link design**, where pupils propose the optimum location of a bypass using data used by the Highways agency.

A sample solution

Pupils should quickly notice that it is impossible to use all four tables simultaneously as there are only seven players. On each occasion, therefore someone has to rest. One possible way of organising the matches is shown below.

Start time	Table 1	Table 2	Table 3	
1.00	AvB	CvD	EvF	G rests
1.30	CvA	EvB	GvD	F rests
2.00	EvC	GvA	FvB	D rests
2.30	GvE	FvC	DvA	B rests
3.00	FvG	DvE	BvC	A rests
3.30	DvF	BvG	AvE	C rests
4.00	BvD	AvF	CvG	E rests

This solution was obtained by writing all the players' names on scraps of paper and placing them next to the three tables as shown. Every half an hour the players move one place clockwise. In this way each player plays against all the others once. It is also 'fair' in other ways; each player plays on each side of each table exactly once. Notice also that if there were 8 players, the matches would not take any longer. The additional player could play the resting player.



3 Notes on the unstructured problems (continued)

Designing and testing **Designing a box for 18 sweets**

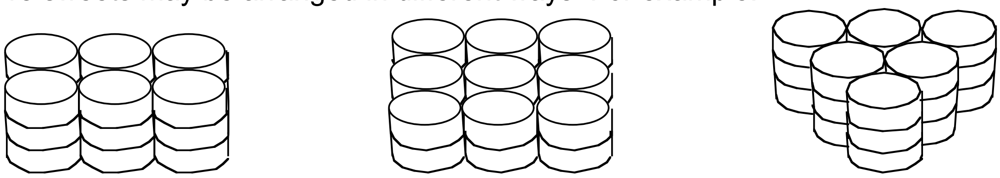
Links to Case Studies

This problem involves designing and making a product subject to given constraints (18 sweets, A4 card). The product is then tested for suitability. This is similar to several problems within the Case Studies, most notably:

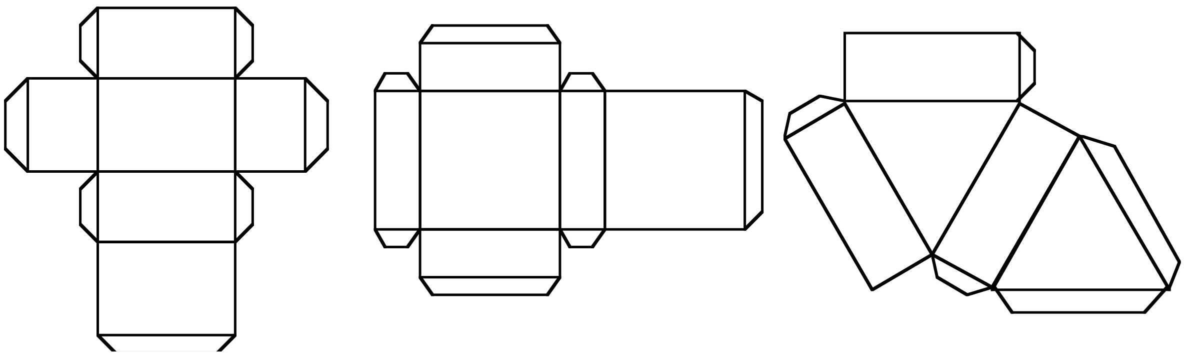
- **Product wars**, where pupils are asked to package a drink and test it through market research.
- **Digi design,** where pupils create a graphical design for a character and test it through market research.

A sample solution

18 sweets may be arranged in different ways. For example:



Each arrangement will lead to a different box design. Their dimensions may be calculated theoretically, or a more concrete approach may be adopted by drawing round sweets with appropriate dimensions[1]. Furthermore any given design may be constructed from card in several different ways. Some possible box designs are illustrated below:



[1] Fruit pastels and wine gums are commonly sold with these dimensions.

3 Notes on the unstructured problems (continued)

Exploring and discovering **Calculating Body Mass Index**

Links to Case Studies

This problem is concerned with the systematic controlling of variables in order to find underlying relationships in a real situation and to use these relationships to make predictions and check the model. This is similar to several problems within the Case Studies, most notably:

- **Crash test,** where pupils systematically explore the effect of different variables when crash testing cars.
- **Speed cameras**, where pupils systematically investigate the effects of different sites for speed cameras.

A sample solution

It is easy to find the boundaries at which someone becomes underweight/overweight/obese if one variable is held constant while the other is varied systematically. The boundaries occur at:

	BMI
Underweight	Below 18.5
Ideal weight	18.5 - 24.9
Overweight	25.0 - 29.9
Obesity	30.0 and Above

In order to find out how the calculator works, it is better to forget realistic values for height and weight and simply hold one variable constant while changing the other systematically. For example, if pupils hold the height constant at 2 metres (not worrying if this is realistic!), then they will obtain the following table and/or graph:

Weight (kg)	60	70	80	90	100	110	120	130
BMI	15	17.5	20	22.5	25	27.5	30	32.5
	Underweight		Ideal weight		Overweight		Obese	

From this it can be seen that there is a proportional relationship between weight and BMI. (If you double weight, you double BMI; Here BMI = Weight/4)

If they now hold the weight constant and double the height, they will see that the BMI decreases by a factor of 4. This is an inverse square law, which may be outside the experience of many pupils. They may be able to explore the relationship by graphing, however.

So, if the BMI is proportional to weight and inversly proportional to the square of the height, it makes sense to try the relationship BMI = k x (weight)/(height)^2. The result is that k = 1.

4 Practical advice for teaching problem solving

Allow pupils time to understand and engage with the problem Discourage pupils from rushing in too quickly or from asking you to help too soon.	*Take you time, don't rush.* *What do you know?* *What are you trying to do?* *What is fixed? What can be changed?* *Don't ask for help too quickly - try to think it out between you.*
Offer strategic rather than technical hints Avoid simplifying problems for pupils by breaking it down into steps.	*How could you get started on this problem?* *What have you tried so far?* *Can you try a specific example?* *How can you be systematic here?* *Can you think of a helpful representation?*
Encourage pupils to consider alternative methods and approaches Encourage pupils to compare their own methods.	*Is there another way of doing this?* *Describe your method to the rest of the group* *Which of these two methods do you prefer and why?*
Encourage explanation Make pupils do the reasoning, and encourage them to explain to one another.	*Can you explain your method?* *Can you explain that again differently?* *Can you put what Sarah just said into your own words?* *Can you write that down?*
Model thinking and powerful methods When pupils have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	*Now I'm going to try this problem myself, thinking aloud.* *I might make some mistakes here - try to spot them for me.* *This is one way of improving the solution.*

5 Suggested further reading

A way of thinking about what it means to function mathematically rather than to use mathematics functionally.
Cain, D. (2007) 'Functioning Mathematically: 1' *Mathematics Teaching,* 203, pp8-10
http://www.atm.org.uk/mt/archive/mt203files/ATM-MT203-08-10.pdf

Are all investigations about spotting patterns or are we missing rich sources of mathematical thinking?
Hewitt, D (1992) 'Train spotters paradise', *Mathematics Teaching*, 140, pp 6-8
http://www.atm.org.uk/mt/archive/mt193files/ATM-MT140-p6-8-mo.pdf

The seminal text for asking pupils to think mathematically
Mason, J., Burton, L. and Stacey, K. (1982) *Thinking Mathematically,* London: Addison-Wesley

The book that inspired so much of the research into problem solving heuristics (or what to do when you are 'stuck')
Polya, G. (1957) *How to Solve It: A New Aspect of Mathematical Method*, (2nd Ed) Penguin Science.

Looking at "World Class Tests" and what they show about pupils' strategies when they are stuck.
Pool, P. (2003) 'What do you do when you don't know what to do?' *Mathematics Teaching*, 182, pp 42-44
http://www.atm.org.uk/mt/archive/mt182files/ATM-MT182-42-44-mo.pdf

Can lower attainers exhibit higher-order thinking? Yes, if we ask them to!
Watson A. (2001) 'Low attainers exhibiting higher-order mathematical thinking' *Support for Learning* 16(4) Nov pp.179-183
http://www.education.ox.ac.uk/uploaded/annewatson/watsonsocjustlowattainers.pdf