BOWLAND
MATHS

How Risky Is Life?

A Mathematics Case Study

Teacher's Guide

Shell Centre
for Mathematical Education

FOREWORD

This case study is designed to help develop real problem solving in mathematics. It is intended for a class of KS3 pupils working with a teacher. We hope you find that the work contained within this case study provides interest and support for all pupils, yet still has the potential to challenge high attainers.

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The case study has been developed in classrooms with teachers and their pupils. We are grateful to them all. The feedback from classroom observation, the comments of teachers and pupils, and an analysis of their work have enabled the materials to be developed and refined.

We welcome further comments, particularly from those who have used the materials in class. Please forward your comments to Shell.Centre@nottingham.ac.uk

We hope you and your pupils enjoy this case study, as others have.

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CONTENTS

Conventions used

Throughout these materials different type faces and conventions are used.

Text like this is "teacher talk".
This text illustrates the kind of thing the teacher might say to the pupils.(or, occasionally, pupils to each other and to the class)

> Boxed text indicates opportunities that arise for working further on the mathematics**.**
>
> You may not wish to pursue these immediately, but they may be suitable to raise with more capable pupils..

INTRODUCTION

"My parents won't let me go out on my own. They think I'll be mugged, or run over."

Overview

This case study tackles something that affects (and impoverishes) people's lives, liberties and happiness – the mismatch between real and perceived risk. Pupils explore the risks of dying unexpectedly from various causes. They start from the fears they know and, by comparing them with real-life data, they recognise that these are often unfounded and driven by the media. Pupils learn how to calculate the risks involved for various activities and how these are related to the base risk of death for typical people of different ages and genders. The emphasis is on order-of-magnitude comparisons, reflecting the various kinds of variation in risk level between individuals and over time.

Pupils learn, in a down-to-earth way, that:

- Life is risky, but not very – that, for people in our society, the probability of any unexpected death to them or their loved ones is tiny, but not zero;
- Mathematical thinking is essential for getting risks in perspective, as a first stage in accepting that small risks are an inevitable part of life – and that it is orders of magnitude, rather than precise numbers, that are significant;
- The media do stories rather than information. They know that a vivid story grabs people's attention; they usually don't worry that the implications may be misleading. While the chances of X happening to any individual are very small, if we take 50 million people, the chance that it happens to someone may be high. The media focuses on the someone and implies that, because it happened to them, it is likely to happen to you. Life decisions need to balance risk against benefits, based on evidence.

Mathematical content

This case study addresses most of the Key Concepts and Key Processes in the KS3 Programme of Study. Pupils are expected to represent a context from the real world, analyse it using mathematical procedures, interpret and evaluate the evidence and communicate and reflect on their results. As with all substantial problems, the work involves pupils selecting, organising and using concepts and skills they have been taught, as well as exploring new ones. The KS3 topics most featured are:

- **Number and Algebra**: Using rational numbers, their properties and their different representations; Using and applying ratio and proportion; accuracy and rounding.
- **Statistics:** Applying the handling data cycle; Using measures of central tendency and spread; experimental and theoretical probabilities.

In addition, pupils are offered opportunities to: interpret very large and very small numbers; to use appropriate orders of magnitude; to use a range of representations, including tables and graphs of data, probabilities and distributions; to explore random variation and statistical inference from data.

Organisation and pedagogy

This case study supports 5-6 one-hour lessons of classroom activity, interspersed with modest amounts of homework, either on successive days or more spread out. It contains a teacher's guide, pupil materials, and computer software. It is most suitable for pupils in Years 8 or 9. Stage 4 is more challenging mathematically than the first three, but is designed to be valuable in different ways to pupils at different levels. The issues dealt with in this case study may be sensitive to some pupils. It may be helpful to discuss the approach to be taken with PSHE colleagues and/or with teachers of English and other subjects where sensitive issues commonly arise. The sequence of activities is as follows (the timings are only indicative):

- **Stage 1: Exploring our perceptions of risk** (1 hour) Pupils discuss various types of unexpected hazard that concern and worry them, their friends and their family. The focus moves to potentially fatal hazards. Groups rank different causes of unexpected death in order of risk, then try to estimate the numbers in the whole population that will die in a year from each cause.
- **Stage 2: Discovering the facts** (1 hour) Pupils are given the numbers of people who died in 2005[1] from these causes. They compare them with their perceptions, going on to calculate what proportions of the total population each number represents and display these 'odds' in various ways. It emerges that unexpected deaths are very unlikely.
- **Session 3: How risky is life? The big picture.** (1 hour) Attention now focuses on 'base' probability, the total risk of dying in a year. This is dominated by illness-related deaths, which are strongly age-dependent. The overall picture is built up from further data on various causes of death in various age bands.
- **Session 4: Do these estimates apply to me?** (1 hour) Attention moves to two deeper questions, exploring variation and prediction. Is 2005 data relevant for future years? Is population data relevant for me? A computer simulation supports an exploration of the random variation that may be expected in the data.

A mixture of class, group and individual work is involved, as in the real world. Your role is to set pupils realistic targets, challenge pupils to think and reason for themselves, and manage discussions and plenary reporting sessions. You should only demonstrate techniques as a last resort. Throughout the goal is to develop pupils' ability to work and think independently.

Resources

The resources provided in this case study are as follows:

- **A teacher's guide**. This contains the suggested lesson plans.
- **Pupil handouts**. The 12 handouts should be photocopied and distributed to pupils as outlined at the beginning of each stage in the teacher's handbook.
- **The software.** For Session 4, a simple computer simulation is provided to enable the class to explore random variation. This software can be used by groups of pupils or simply by the teacher on the IWB in class discussion.

In Stages 1 and 2 each group of pupils will also need: A large sheet of A3 paper, a glue stick and a felt-tipped pen. Calculators will be needed throughout. In addition, a data projector or interactive whiteboard is useful for the classroom presentations.

[1] This is the most up to date data available at the time of writing.

STAGE 1: EXPLORING OUR PERCEPTIONS OF RISK

Aim of this session

- Explore various risks; focusing on causes of unexpected death;
- Estimate which causes are most likely;
- Estimate the numbers of people from the total population that will die in one year from each cause;

This will involve:
- thinking in terms of orders of magnitude;
- handling large numbers.

Time needed

About one hour

Resources needed

A few days before, you may like to ask pupils to look at their newspapers at home and to collect a few examples where the media seem to be encouraging a 'climate of fear'. (Alternatively, this may be done as homework after the session)

Pupils should be grouped into twos or threes. Each group of pupils will need copies of the following sheets.

S1: *Some press cuttings*
S2: *Brainstorming sheet*
S3: *Possible causes of unexpected death (cut out)*

For making posters, each group of pupils will also need
- a large sheet of A3 paper,
- a glue stick,
- a felt-tipped pen, and
- a calculator.

There are two different ways of running the final discussion in this session. You may enjoy using either: *The washing line approach*,
in which case you will need
- a large copy of each card (A4 size) on sheets S3;
- a washing line;
- some clothes pegs for attaching the cards to the line.

or you may choose a more conventional approach in which you organise the suggestions on the board.

Suggested sequence of activities

Introduction: Brainstorming risks in groups

Organise the class in groups of three or four (pairs are less good for this) and give each group a copy of S1: *Some press cuttings* and S2: *Brainstorming sheet*.

Introduce the idea of risk to the whole class:

> *We hear a lot about the dangers we face in everyday life.*
> *Life seems to be a risky business.*
> *What dangers do we face every day?*
> *What things do you, your friends, your family, actually worry about?*
> *How do we change our lives as a result?*

Ask groups to consider these questions and to list their thoughts.

Whole class discussion: Collecting and sharing ideas

Collect ideas from the class and list these on the board. List the dangers on the board under three columns headed 'Serious', 'Very serious', 'Life or death' in discussion with the class.

The following list was obtained from one class:

Serious	Very serious	Life or death
• Breaking up with girlfriend/boyfriend • Spiders • Wasps • Getting old • Not passing exams/ SATS • Family fallouts • Dentists	• Bullying • Having an accident • Money problems • Getting ill • Parents getting divorced • People speeding • Being sent to a foster home	• Death of self • Death of family • Car crash • Having a heart attack • Global warming • Getting cancer • Being attacked • terrorist attack • rail crash

Now encourage pupils to write down some ways in which people's fears affect their everyday lives. For example:

> *My parents won't let me walk anywhere on my own.*
> *I'm not allowed to ride a bike to school.*
> *My dad is giving up smoking.*
> *We are made to eat fresh fruit instead of crisps.*
> *We have to 'cover up' in the sun.*
> *People have to practise 'safe sex'.*
> *I have to wear a bike helmet.*
> *My mum goes to the gym once a week.*
> *My uncle refuses to fly anywhere.*
> *We didn't dare go into central London after the terrorist attack.*
> *My gran is afraid of being mugged so she won't go out anywhere.*

Conclude this part of the session with the questions:

> *Which fears do you think are real and which are exaggerated?*
> *Do we reduce the risks we face very much if we change our behaviour?*
> *Do the changes we make to our lives 'spoil' our enjoyment of life?*
> *These are the kinds of question we will consider as we work on this case study.*

Small group work: Comparing different risks

It is important to narrow the focus (risk is a huge subject!).

> *To begin with we'll focus on unexpected 'life or death' risks.*
> *How big are these risks?*
> *Are some of them exaggerated?*
> *First we'll list these causes and estimate how big these risks are.*

Ask pupils to work in groups of two or three.
Give each group S3: *Some possible causes of unexpected death.*
These should be cut into cards so that they may be sorted.

> *Place the possible causes of death in rank order, with what you think is the most common cause of death at the top and the least common at the bottom.*
> *Try to agree on your ordering.*
> *There are extra blank cards to write in other causes you may think of.*

Small group work: Estimating numbers that die

Allow groups a few minutes to do this.

> *The total number of people in England is about 50 million.*
> *How many people in England do you think die from each cause in a year?*
> *1? 10? 100? 1000? 10,000?*
> *Which number will be nearest?*
>
> *Of course, you probably don't know, but have a guess and write your answer on the card.*
> *These are only very rough estimates.*

> **Orders of magnitude**
> You may like to introduce the terminology 'orders of magnitude' here.
> We are only interested in estimating to an appropriate order of magnitude.

Give each group the materials for their poster. When pupils have agreed on a rank order for their risks, they should stick the cards down onto the poster in order (use the whole height of the poster). They should then write on each card their rough estimate of the number who die each year from that cause.

Small group work: Estimating proportions that die (optional, challenging)

If there is time, (*make sure you leave time for the final whole class discussion*), ask pupils to calculate the proportions of the total population that die from each cause, according to their estimates. (*This is challenging for most pupils; we shall return to it more gently next time*)

> *There are about 50 million people in England.*
> *If you think that about 100 people died in fires, how would you calculate that as a proportion of the total population?*
> *(50,000,000 ÷ 100 = 500,000. So it's about one person in 500,000 each year.)*
> *Is that more or less likely than one in a million?*

Whole class discussion: Sharing perceptions

Get a spokesman from each group in turn to share their estimates.

This may be done as a normal class discussion or, in a lively way, using large versions of the cards and a 'washing line' across the room. Ask one group to peg their cards in order on the line. Successive groups can then come out and modify this order, while explaining why they think that some events are more risky than others.

In either of these modes, the discussion focuses on comparing their estimates of risks.

You seem to think that the risk of a plane crash is greater than that of dying in a road accident. Why is that? Is it because more people die when it does happen?

You seem to think that the risk of being murdered is greater than that of dying from an accidental fall. Why is that? Is it because there is more murder reported in the newspapers?"

Looking at the spread of your views on the risks, it seems we need to see if we can learn more about the facts. We'll move to that in the next stage.

Different representations
You may also wish to discuss different ways of representing the risks.

e.g. using mini whiteboards:
Show me another way of representing a 1 in 5 risk? As a fraction? A decimal? A percentage?

Is this more or less likely than a 1 in 10 risk?

Discuss the reasons why people think that some causes present greater risks than others.

You may want to end the session on a reassuring note:

So now we've made our rough estimates.
Next time we'll begin to look at the facts.
You'll find that life is a lot less risky than you think.

Possible homework: Collecting examples from the media

Ask pupils to look at their newspapers at home and to collect some more examples where the media might be encouraging a 'climate of fear'.

STAGE 2: DISCOVERING THE FACTS

Aim of this session

To compare pupils' beliefs about the relative risks with the facts.

Time needed

About one hour

Resources needed

Each pupil will need:

- a calculator.

Each group of pupils will need:

- their posters from Session 1, and
- a copy of the following sheets:
  S4: *Number of (mostly) unexpected deaths in 2005 (England and Wales)*
  S5: *Estimated proportion of people that die from each cause.*
  S6: *Representing the proportions*

For making new posters, each group of pupils will also need

- a large sheet of A3 paper,
- a glue stick and
- a felt-tipped pen
- a calculator.

Suggested sequence of activities

Introduction: Reviewing and sharing

Begin by sharing a few of the newspaper clippings gathered by the class since the last session.

Remind the class of the likelihood of different dangers people face every day.

> *Some of the fears we looked at in the previous session are much more likely to happen than others. There seems little sense in worrying about events that will almost never happen. In this session, we will look at the facts.*
>
> *Now I am going to give you a collection of cards showing the same hazards as before (S4). This time, on each card, there is a number which tells you how many people died of this cause during one year.*
> *Place the cards in order, again with the most frequent first, and compare your new rank order with what you predicted in the last session.*

Working in groups: Comparing perceptions with the facts

Give each group a copy of sheet *S4: Number of deaths from various causes in 2005* Allow pupils time to cut out and order these causes, compare them with their poster, and discuss their findings.

As they do this give each group a calculator and the card set *S5: Estimated proportion of people that die from each cause*

Working in groups: Relating proportions to numbers

Suggest that pupils write on each strip on S5 how many people are represented. (*It is easier of they do this in sequence, the risk decreasing by a factor of 10 each time)*

> *There are about 50 million people in England.*
> *So how many would 1 in 10 be? (5 million)*
> *I in 100? (500,000)*
> *Is that more or less likely than 1 in 10?*
> *Work out and write the appropriate number on each strip.*

As they do this give each group the materials to make a new poster.

Working in groups: Calculating the size of these risks

With the numbers written on the strips, they will find it straightforward to place the S4 cards between these strips by comparing numbers. They should obtain a result that looks a little like the poster shown on the next page.

A successfully completed poster

What is the risk of dying from...

Low risk	**1 in 10,000** One death in ten thousand people each year	Approx. number out of 50 million people 5,000
	Accidents at home 3,892	
	Road accidents 2,697	
	Accidental poisoning 913	
	Assault and Murder 643	
Very low risk	**1 in 100,000** One death in a hundred thousand people each year	Approx. number out of 50 million people 500
	Accidents at work 350	
	Forces of nature (Storms etc) 86	
	Terrorism 51	
Minimal risk	**1 in 1,000,000** 1 death in a million people each year	Approx. number out of 50 million people 50
	Air travel accidents 19	
Almost no risk at all	**1 in 10,000,000** 1 death in ten million people each year	Approx. number out of 50 million people 5
	Accidents at school 4	

(*Optional, for fast-moving groups*) Ask pupils to calculate the proportions of the total population that died from each cause in 2005, using the actual numbers of deaths. (*Most will find this challenging*)

643 people out of 50 million died from assault and murder that year.
That's the same as saying one person in ….. how many?
(One in 50,000,000÷643 ~ 1 in 80,000)

It is preferable to allow pupils to discover their own method, if they are capable.

Standard form
If pupils calculate the proportion as 643÷50,000,000 on many scientific calculators they will get: 1.286E-5
This is a possible opportunity to introduce standard form, and appropriate accuracy.

Whole class discussion: Discussing the posters

Ask pupils to describe what they found most surprising when making the posters. Remind them of the newspaper clippings.

Which risks do people worry too much about?
Which risks do you think people worry too little about?

Whole class discussion: Representing the proportions visually

Give pupils copies of sheet S6: *Representing the proportions*.
(If need be pupils can refer back to the strips on the posters – but better without)

How many big squares are there altogether? (100)
If there are 50 million people, how many is that for each big square?
(50,000,000÷100=500,000)
How many tiny squares are there altogether? (100 x 100 = 10,000)
If there are 50 million people, how many is that for each tiny square?
(50,000,000÷10,000=5,000)
Now shade in the diagram to show how many people die from the different causes.
Some of the causes are labeled; shade in near the label for that cause.
(road accidents – half a tiny square; all the others less ~ little dots.)
Not much is it?

This diagram vividly shows that in any given year the chances of dying from an unexpected cause is very small indeed! You should hardly see the shading.

Now begin to relate this to the bigger picture

Are these really all the deaths?
We saw that there are about 50 million people in England.
Can you make a reasoned estimate of the total number of people that die in each year?

Ask pupils to spend a few minutes discussing this question, in pairs. Then share estimates and reasons.

If everyone lived for 80 years, you might expect about 1/80th of the population to die each year. This would give an answer of about 50,000,000÷80 = 625,000.
This is a pretty good estimate. In fact about 500,000 people die each year.
(That's not because we all live to 100; the population is expanding with more young people)

The deaths we have been looking at are nothing like as big as this number.
So, where do all the other deaths come from? (Illness, disease).
Think about this before the next session, when we will look at other causes.

Keep your diagrams S6. We'll need them next time.

STAGE 3: HOW RISKY IS LIFE? THE BIG PICTURE.

Aim of this session

To recognise that:

- most deaths are from illness
- this risk is strongly age-dependent

To look more closely at age and gender-related data to see the

- total "base risk" as a function of age
- balance of different causes in different age ranges.

Time needed

About one hour.

Resources needed

Each group of pupils will need:

- The S6 diagrams from Session 2,
- a calculator, and
- a copy of the following sheets:
  S7: *How risks vary with age*
  S8: *Bar graphs (cut up)*
  S9: *More bar graphs (cut up)*
  S10: *Pie charts (cut up)*
  S11: *True, False, or Impossible to tell? (cut up)*

If possible, O1: *Base risk of death by age* should be available for display on OHP or whiteboard/data projector. An on-screen version can be found in the case study download. Otherwise, give each group a paper copy.

Introduction: ***How "base risk" varies with age and gender.***

Link to last session

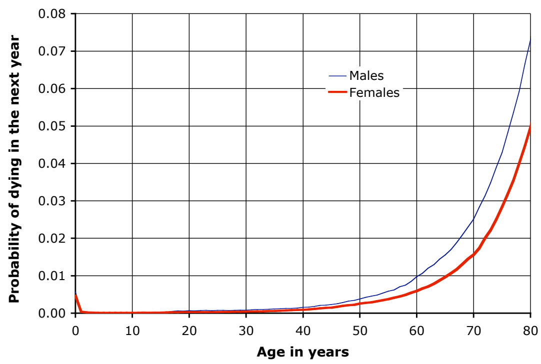
Last time we finished with a bit of a puzzle.
If we add up all the causes of unexpected death on sheet S4, we can see that only about 10,000 people die each year from all the causes that people worry so much about.
Yet we estimated that about 500,000 people die each year
So what do you think that most people die of? (illnesses)

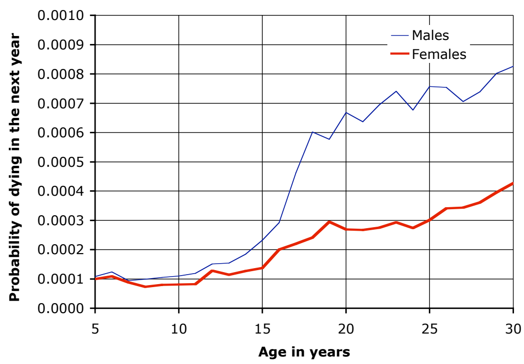
Let's look back at our chart (S6) with the squares representing how many died.
What will 500,000 deaths from illness look like on that chart?
Fill that in on a different big square and label it

Leave time for groups to shade in 500,000 (one big square) on their S6 charts and label it 'Illness'

Some of you may be thinking:
'Yes that is all very well, but surely most of these deaths happen to older people than me."
Now let's look at this.

Show each of the following two graphs (O1) in turn (on the IWB or OHP) and ask pupils to describe what they show.





Both graphs show how the chance of dying in the next year varies with age.
This is called your "base risk of death".
This information is used, for example when calculating life insurance payments.

So what is the difference between these graphs?
(They are showing different age ranges. In fact the second is an enlarged part of the first).

Look at the second graph. What is the probability for, say, 10 year olds? (about 0.0001)
That means "1 in how many"? (10,000 – you may want to review this with other decimals, 0.01 etc)

What are the main messages in the first graph?
(The chance of dying before the age of about 60 is very small indeed.
The base risk is less than 0.01, which means that fewer than one person in one hundred dies each year. After the age of 60, the probability increases quite rapidly.
Males have a greater chance of dying than females throughout their lives.)

What are the main messages in the second graph?
(After the age of 15, males have a much greater chance of dying than females.
This is probably not due to illness, as they are still young, but rather due to the fact that males live more risky lives! Even so, this risk is very small.)

The purpose of showing these graphs is to illustrate the fact that risks depend on two variables: age and gender. In this session, we will explore how different causes of death depend on these two variables.

Introduction to the main activity: Learning from the data

Give each pupil a calculator and a copy of the data on causes of death (S7) (they need one each because this data is quite detailed and rich). Also give each *group* of pupils an envelope containing the bar graphs (S8, S9) and pie charts (S10).

Ask a few straightforward questions that will help pupils to interpret the tables on S7. You may need to help pupils to express the large numbers in words. Often it helps to round them.

How many men aged between 60 and 79 died in an accident? (1,235)
What is this figure to the nearest hundred? (1,200)
How many males are there in England and Wales aged between 1 and 19? (6,390,600)
What is this figure to the nearest hundred thousand? (6,400,000 or 6.4 million)

You may also like to try asking some more challenging questions that involve calculations. For example, you could refer again to "base risk".

How many people aged between 1 and 19 died in one year? (2,327)
What is this figure to the nearest thousand? (2,000)
About how many people aged between 1 and 19 live in England and Wales? (12,455,300)
What is this to the nearest million? (12,000,000, ie 12 million)
So what proportion dies in one year? About 1 person in 6? 1 person in 60? 1 person in 600? ... or what? (1 in 6,000 ~ 2,000/12,000,000)

Now repeat this for the over 80s. (264,915/2,384,800 ~ 265,000/2,400,000 ~ 1 person in 9 per year).
That is much greater.
If all of us in this class were over 80, how many would we expect to die in one year? (about 3 people).

Working in groups: What do the bar graphs show?

Ask pupils to work together to work out *from the tables* what each bar graph shows. Each time, they should write a title on the graph.

(*For fast-moving groups)* Ask pupils to work out what each pie chart shows. The correct answers are:

Bar graphs						**Pie charts** (optional)			
A	B	C	D	E	F	A	B	C	D
Total population	Total illness	Total accidents	Road accidents	Falls	Assault and Murder	Age 20-39	Age 1-19	Age 60-79	Age 40-59

All groups may well not complete this work. **Make sure** there is time for them to tackle (not necessarily complete) the final *True, false or cannot tell?* activity. Give out S11: *Statement cards.*

Working in groups: ***Evaluating statements - True, false or cannot tell?***

Refer to the S11: *Statement cards*.

> *For each statement, I want you to work together to decide whether you think the statement is true, false or whether it is impossible to tell without being given more data.*
>
> *Try to find convincing reasons using the data and write these reasons down. I will ask each group to report back on their reasoning to the whole class.* ***The reasons must refer to the data in the graphs or the tables.***

Make it clear that they should use the data on S7 and/or the graphs.

Whole class discussion: ***Sharing reasoning on the main causes***

Ask each group to share their reasoning on the statements (S11). For example:

	Statement	**Typical reasoning**
A	Men live longer than women.	**False** Graph A shows that there are a similar number of men and women under 60, there are more women than men over the age of 60, and almost twice as many over the age of 80.
B	Nearly all deaths are caused by illness.	**True** Graph B (deaths due to illness) would almost fit the total deaths data. In fact 493,066 deaths are caused by illness, out of a total of 509,433. That is about 97%.
C	Road accidents are the greatest cause of accidental death for the under 40s.	**True** There were 1481 fatal road accidents among the under 40s. This is about 60% of all accidental deaths in that age group.
D	Falls are the greatest cause of accidental death among the over 40s.	**True** There were 2882 accidental deaths from falls among the over 40s. This is 34% of all accidental deaths in that age group.
E	Middle-aged men are the most at risk of being assaulted or murdered	**False** The greatest risk is among 20 to 39 year old men where there were 215 deaths. This is the greatest risk even taking into account the different size populations.
F	For the 'over 80s', women are about twice as likely as men to die in an accident.	**False** Although about twice as many females die of accidents in this age group, this is mainly because there are almost twice as many females alive at this age.
G	Men are more dangerous drivers than women.	**Impossible to tell** It is true that men have far more road accidents than females, but this may be due to many causes. (e.g.There are more male drivers, males drive further etc)
H	Old people need not worry about being murdered.	**True** Only 18 'over 80s' were murdered out of a total population of over two million.
I	About 1 in 20,000 of the 'under 20s' dies in an accident each year.	**True** About 626 people out of 12.5 million die in an accident in this age group.
J	About 1 in 5,000 of the 'over 80s' dies in an accident each year	**False** 4688 'over 80s' died in accidents, out of a total population of 2,384,800 million. That is 5000 out of 2.5 million or about 1 in 500.

STAGE 4: DO THESE ESTIMATES APPLY TO ME?

Aim of this session

In this session we explore in a little more depth three important concepts:

- random variation;
- inferences from population data;
- best estimates from the data we have.

The data that has been used in previous sessions is 'old' data from just one year. In addition, this data applies to a large population that includes people who "aren't like me". This raises two important questions:

> How far does this data still apply *now*, several years later?
> How far does this data apply to *me*?

The aim is that pupils come to see that this data provides *best estimates from the data we have.*

We discuss the idea that data will vary from year to year for two reasons.

- There are random 'chance' fluctuations in data that we can do nothing about. This session will help us to estimate the amount of random change we should expect from year to year and get a feel for how this depends on the number that die each year.

- There are underlying changes we, or society, can make to our behaviour that change the likelihood of us dying from particular causes. For example, we may decide to lower speed limits,or to stop smoking.

This session aims to inform pupils' awareness of these issues, not to attempt a full analysis. It is not essential for them to understand all the details of the simulation used. If they engage with the issues, and leave the case study having thought more rationally about the risks they face every day, then we have achieved our aim.

Time needed

About one hour.

Resources needed

IWB and, preferably, one computer per group with the provided computer software "*Simulating random variations*".

Each pair of pupils will need:

- A calculator, and
- the sheet S12: Headlines

Suggested sequence of activities

Whole class discussion: Why does data fluctuate from year to year?

Introduce the following question:

> *In 2005, the population of this country was 50 million.*
> *There were 643 murders*
> *How many murders would you expect there to be in this coming year? 643? Why or why not?*
> *Write down your estimate and give a reason.*

Ask students to discuss this question in pairs. Collect some responses on the IWB:

Estimate	Reason
650	It will be about the same.
700	The trend is rising.
You can't tell	It is just random chance.

Ask pupils to vote with a show of hands on the range of answers they would expect.

> *How close to 643 will next year's number be?*
> *How many people think that it will lie:*
>
> > *between 200 and 1100? (You could bet your lfe on it?)*
> > *between 400 and 900? (Almost certain?)*
> > *between 600 and 700? (Very likely?)*
> > *between 630 and 660? (Likely?)*
> > *between 642 and 644? (Unlikely?)*
>
> *What about the year after? (Same answer?)*
> *Why do you think the numbers fluctuate from year to year?*

Encourage discussion of the possible reasons for fluctuations in the data. The following two reasons should emerge quite naturally from the class:

> ***Changes in the situation***
> *The number of murders may increase if guns become more commonplace.*
> *The number of murders will go down if police catch the main culprits.*
> *If they crack down on other crime, like drugs, then murders may reduce as well.*
>
> *In using data from the past for prediction, you need to be vigilant – think about how the situation may have changed and how big an effect this may have had.*
>
> ***Random fluctuations***
> *Even when nothing has changed, there will be some random variation.*
> *It is important to know this. If the newspaper says "Murder on the increase" we want to know whether this is likely to be random fluctuation or whether there is an underlying cause.*
> *Let's explore the size of fluctuations that we should expect.*

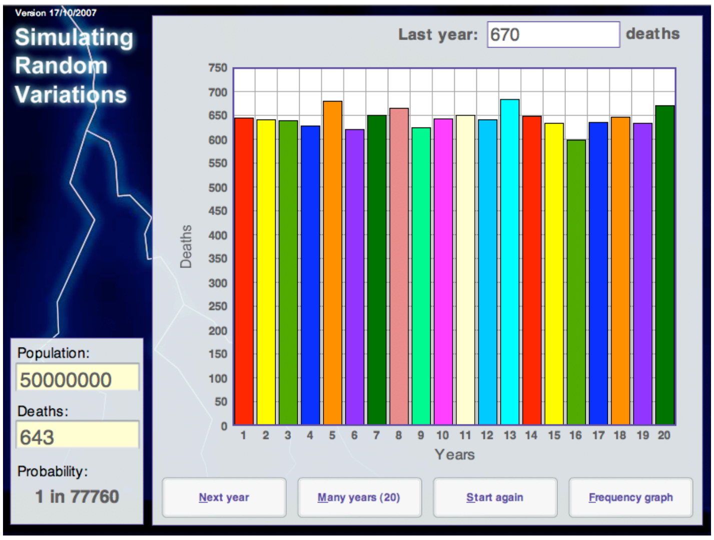
Explain that these will be addressed in the rest of the session.

Whole class discussion: Random fluctuations

Introduce the computer software *"Simulating random variations"* to the class on the interactive whiteboard. In the "Population" box, enter the total population, 50,000,000 and in the "Deaths" box enter 643. Point out that, automatically, the computer shows the proportion of people that were murdered (1 in 77760). Ask where this number came from. (50,000,000÷643).

This program shows what happens with just random variation from year to year.

Click on the "Next year" button several times and explain that, as you do so, the computer is running a simulation to see what the expected number of murders will be. (It is not making *predictions* here, it is just simulating a random process where the probability of something happening is 1 in 77760.)



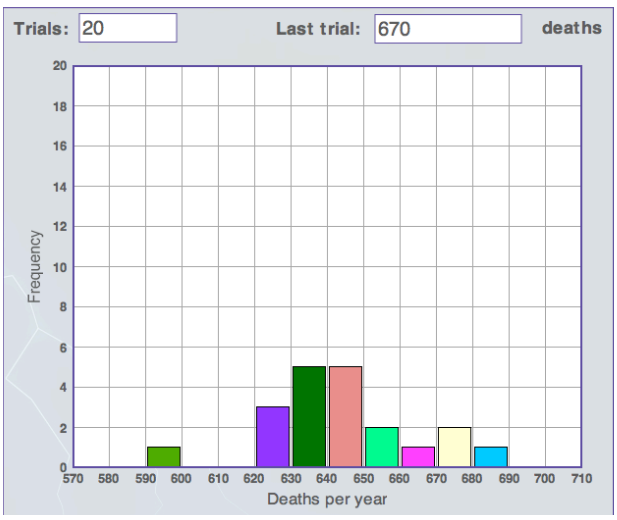
Simulating the situation
You may like to explain the idea of a computer simulation as follows:

Imagine that you have 50 million pieces of paper in a hat and 643 of them say M ('murdered'). What the computer is doing is like taking a piece of paper out of the hat, writing down what it gets, and then putting it back. It does this 50 million times and sees how many M's it gets.

As you click, read out the numbers of deaths that you obtain:

649, 600, 630, 670, 660, ...
What kind of variation are we getting here?
Most of the answers seem to be around 620 to 660, but there are a few outside of this.

(*Optional*:) At this point you may choose to show the class the **Frequency graph** showing the number of times each number of deaths was obtained.



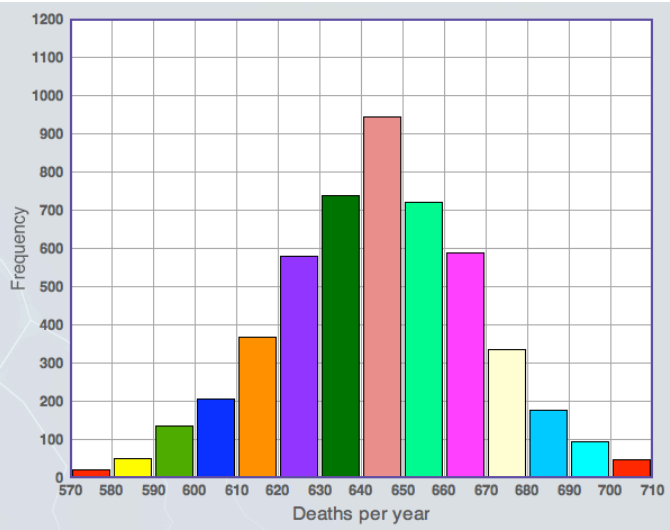
What kind of variation are we getting here?
Most of the answers seem to be around 620 to 680, but there are a few outside of this.

Theoretically
Random variation is expected to give numbers within about 25 of 643, i.e. mostly in the range
620 to 670.

In general, for small n, the random variation should be about $n \pm \sqrt{n}$.
(This assumes a normal distribution with large *n* and a small probability of the event occurring.)

Click on the "100 more trials" button to see how the frequency graph changes after many more random trials. This will result in the well-known 'bell shaped graph"

This shows the variation when we run the simulation 5,000 times.



Follow this with one of the following two group investigations. The second is more suitable for groups who enjoy a challenge.

Group investigation 1: Newspaper stories

Give each pair of pupils a copy of S12 *Headlines* together with the data on S4: *Number of deaths from various causes in 2005*. Ask them to study each headline and use the software to decide whether the newspaper is trying to create a story out of nothing more than random fluctuations in the data.

WATCH WHAT YOU EAT! The number of deaths from poisoning has risen to 950 - and increase of 50 on the previous year. Doctors are advising us to be more careful when reheating frozen food.	**50% MORE SCHOOL DEATHS** Over the past year, the number of fatal accidents within schools has risen by 50%. "We must introduce more safety measures" said one spokesman. " Schools are becoming increasingly dangerous places."
MURDERS ON THE INCREASE The number of murders in England and Wales increased by 3% last year. "This is just one more depressing statistic on the state of our society said the opposition spokesman."	**FEWER PEOPLE KILLED ON OUR ROADS** Road deaths dropped by 10% in the past year. "This proves that the introduction of speed cameras has been effective." claims a department of transport spokesman.
HOMES BECOMING SAFER The number of fatal accidents in the home fell by about 5% last year claimed the centre for research into household accidents. This shows the effectiveness of our recent campaigns.	**FEWER AIR DEATHS** The number of deaths in air accidents decreased by 20% last year. "This is proof that the safety measures we put into place last year are now working", claimed a spokesman.

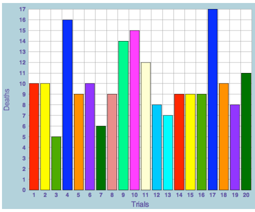
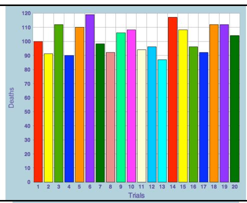
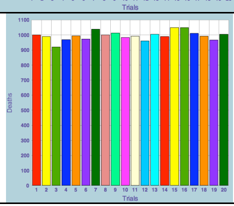
Significance
In almost every scientific study or experiment, the results have to be checked mathematically to make sure that they are **significant**.

When scientists say this, they don't always mean "big" or "important" - just that they are sure that they have measured something real – not a random fluctuation.

Group investigation 2:

How does random variation depend on the number?

Pupils may now work in pairs at computers. Ask the groups to investigate other values of *n*, (for example: 10, 100, 1000, 10,000,…) and use the software to estimate the approximate spread of results in each case, recording what they find. For example, they might choose to record their results as follows:

Number of people dying in one year (n)	Simulation of variation	Approximate spread
10		The number of deaths vary from about 5-17 This is a spread of about 12
100		The number of deaths vary from about 90-120 This is a spread of about 30
1000		The number of deaths vary from about 950-1050 This is a spread of about 100

As you go round, encourage pupils to estimate the spread of results by eye (the frequency distribution makes this easier) and ask specific questions such as:

> *Is spread roughly proportional to n?*
> *If you multiply n by 10, does the spread multiply by 10?*
> *(If you multiply n by 10, does the spread multiply by roughly the same factor?)*
> *(Sketch a graph of spread against n if you like)*

Pupils may notice from the simulation bar graphs that the random variation appears less (as a proportion of the total) when large numbers are dying each year. This is because this number is not proportional to *n*, but to its square root – a useful rule – so even more challenging questions could be put to the most capable:

> *More advanced theory says: the fluctuations in n should be roughly the square root of n. Does this fit in with your data?*

If necessary, remind pupils that this is a simulation, using random numbers:

> *Remember, the computer can't really work out how many people will die next year. But, if you are looking at real figures, and see a change, it is very important to be able to know if it could just be a random variation. In any serious research, the results must be carefully checked to make sure that they can't be explained by chance.*

Whole class discussion: ***How far do the predictions from data apply to me?***

The numbers we have been looking at are from the population as a whole.
How far do these probabilities apply to an individual?

*How far do these estimates apply to **you or me**?*

How typical is each of us of the UK population?

Of course, everyone is an individual with their own characteristics – male or female, in good health or not, risk-taking or cautious.

But how far are these differences likely to affect your risk level?
Let's be more specific:

The data suggests that the base risk of dying in the next year for a 15-year old boy is about 1 in 5,000. Is this a reliable estimate of the risk for me?

Points that should emerge include, for unexpected death:

While a cautious approach to life can reduce risks of accidental death, excessive caution (e.g. never going out on your own) has other undesirable effects on health and opportunity to enjoy life.

Being cautious is unlikely to change these risks by an order of magnitude (for more than half the people killed in road accidents, it was not their fault)

and for illness:

Medicine is learning more all the time about the factors that affect chances of disease, whether environmental (pollutants) or genetic (genes that increase chance of breast cancer, more generally, family history is a factor).

Is there any more detailed data that would allow a better estimate?
If so, would it change the estimate enough to make it worth the effort?

Some important data is readily available – male/female and age, and the effects of smoking are the prime examples

Generally speaking, the population data provides a good rough estimate that enables you to:

gain a realistic assessment of the various risks
bear the numbers in mind when taking your life decisions.

Unless we are prepared to find and analyse data for sub-populations, it provides the **best estimate** available.

APPENDIX A: SOFTWARE NEEDED FOR STAGE 4

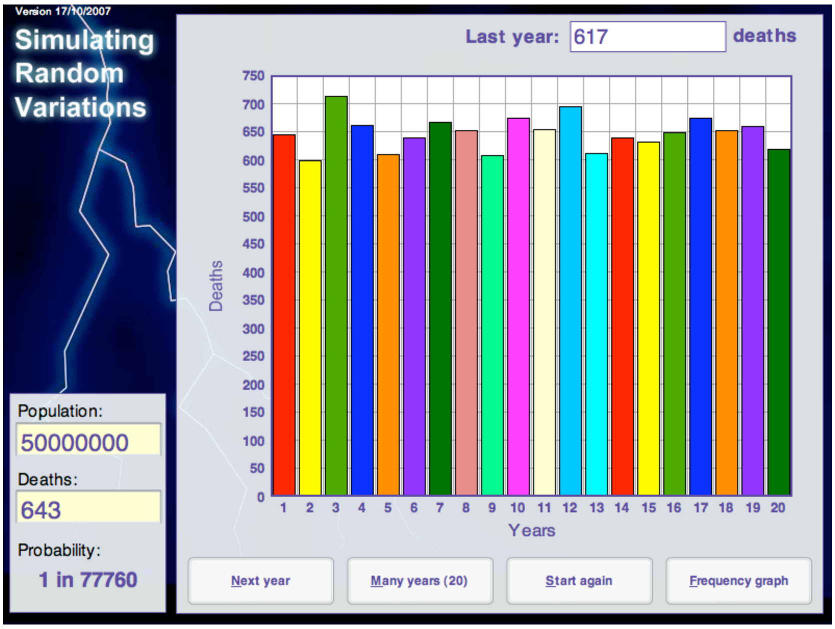
Simulating Random Variations

This software is only required for Stage 4, and is intended for use on an interactive whiteboard or data projector to support a whole class discussion, as well as on individual computers for small group work. The software is available for PC and Mac.

See the "overview" page for this case study on the Bowland DVD for details of how to install and run the software (if you downloaded the case study, double-click on the file "start.htm").

In some cases your IT support person may need to take action to allow the software to work**. Please check in advance that the software works on the actual computers you will be using in the lesson, logged in as a student (if your computers are on a network). Some problems might not show up if you are using a home PC, a teacher's computer or are logged into the network as "staff".**

Contact: Shell.Centre@Nottingham.ac.uk for support



Instructions on how to use the software can be found in the lesson notes for stage 4 of the case study, earlier in this guide.

Mathematical note

The software uses a mathematical model that produces valid results for small probabilities in large populations – the subject area of *How Risky is Life?*

If used to simulate (say) tossing a coin or rolling dice, then the results will be plausible-looking but inaccurate.